



# MECE 308 MECHATRONICS SYSTEM DESIGN I

## OPTIMIZATION

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# INTRODUCTION

If you ask for a definition of "Engineering Design", you should expect different definitions from each person who answers your question. This is because every person has a different job and is interested and specialized in a different field of work. Although they are different verbally, it is possible to find several common points among the given definitions.

One of these common points, perhaps one of the basic features of engineering design, is that **DECISIONS** have to be made during the design procedure.

# INTRODUCTION



In mechatronics engineering design, these are the selection of material or materials and specifying the geometry of some product, machine or machine element. In doing so, design engineers do not have much freedom.

The item to be designed (**design product**) should do a job or more generally, it should serve for predetermined purpose.

# INTRODUCTION

For example, a car engine must develop power from some kind of fuel or energy storage element and a shaft must transmit this power to the drive mechanism or wheels. A toy must be attractive for the children. These considerations make a basis for the decisions and are called **REQUIREMENTS** of the design.

Any decision in a design procedure can not be accepted as valid unless these requirements are satisfied completely. In addition to the requirements, **LIMITATIONS** of design must be also considered in the design process.

# INTRODUCTION

These usually limit the application in two ways; **by the laws of nature and engineering science**, and by the **desirable features of the design product**. The car engine, given above, must develop a certain amount of power, but at the same time, it should be small and light so that it can be placed somewhere within the car body and can carry itself.

An engine which is perfect thermally, but weighs several tons, is not a good solution (**limitation is not satisfied**) for the present day cars.

# INTRODUCTION



The limitation (or one of the limitations) for the toy in the above example, is perhaps that the toy must not be hazardous to the children.

Thus **Power and Weight, or Attractiveness and Hazard** are equally important for decisions of the designer. It is certain that the requirements and limitations (therefore decisions) must be based on acceptable, valid and available scientific and technological principles.

# INTRODUCTION

A "**Decision**" is not a first step nor it is a last step in a design process. It always comes after a mental activity, called **CREATIVITY**. Creativity is usually based on engineering experience and on some previous works which have been found unsuccessful or incomplete at the present stage of development.

# INTRODUCTION

After the design is completed and if it is found successful, the next activity is **MANUFACTURING**. It is the production of the real physical item which has been designed, in required quantities.

Any design procedure usually results in more than one solution and all of these solutions may satisfy the **Requirements** and **Limitations**. Theoretically, number of these solutions is infinite. Each of these solutions (designs) is an alternative design product and is called a **FEASIBLE DESIGN**.

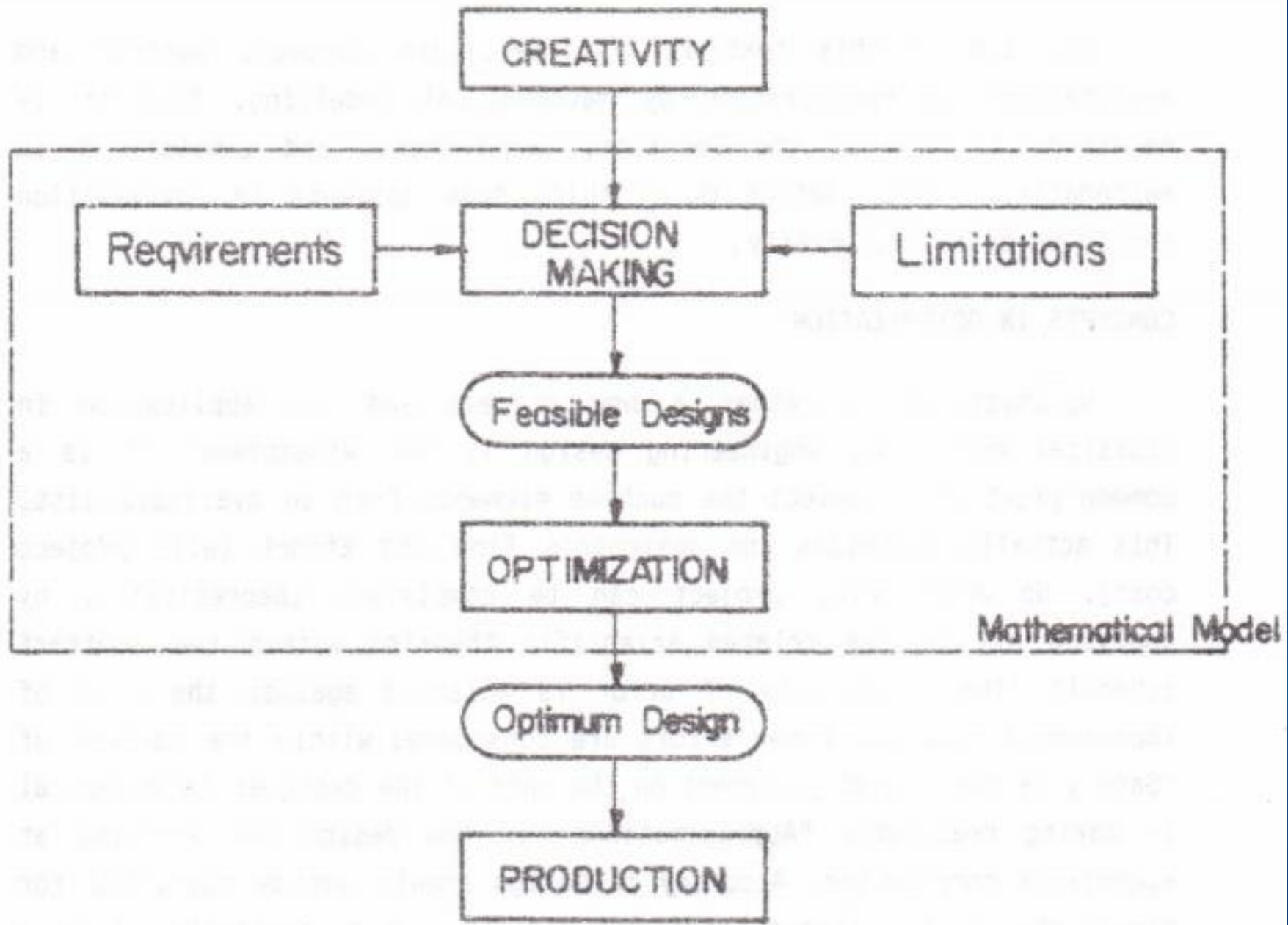


# INTRODUCTION



At this stage, the designer faces the problem of selecting the best of these alternatives i.e., Feasible Designs. The best (Best in a predetermined way) of these feasible designs according to some criterion is called **OPTIMUM DESIGN** and the procedure to obtain that specific solution (optimum design product) is called **OPTIMIZATION**.

The schematic block, diagram in Figure 1 illustrates the place of optimization in a design process and other related activities.



# INTRODUCTION



It is generally necessary to develop a method of **MODELLING** of the final product between Creativity and Manufacturing. The **MODEL** can be physical or mathematical depending on the specific problem and available facilities.

# INTRODUCTION

Usually experimental approaches (**physical modelling**) are expensive and time consuming. It requires production of a physical prototype or model which simulates the real physical system under working conditions similar to the actual case. Although results are more reliable in physical modelling, it is impractical and costly. Mathematical modelling is applied in most of the design problems if the results are sufficient to obtain sound conclusions.

# INTRODUCTION



In the case of optimization, the mathematical modelling is the only applicable method since the production of a number of physical models, slightly differing from each other is not possible at all. Conclusively, optimization should be based on mathematical modelling and it is a paper work. It is always possible to find an exceptional application and certainly, it will be more reliable.

# INTRODUCTION

Our aim in this text is to investigate concept, methods and applications of optimization by mathematical modelling. Thus it is necessary to express the **Decision, Requirements and Limitations** in mathematical forms. Before doing this, some concepts in optimization should be discussed briefly.

# CONCEPTS IN OPTIMIZATION



**Optimization** is rather a new process and its application in classical mechatronics engineering design is not widespread. It is a common practice to select the machine elements from an available list. This actually optimizes the designer's **Time** and **Effort** (also project cost). No engineering project can be completed, theoretically, by applying all of the related scientific theories within the contract schedule.

# CONCEPTS IN OPTIMIZATION

The probability of error is balanced against the cost of improvement time and known errors are considered within the concept of "Safety Factor". Good judgement on the part of the designer is essential in making reasonable "Approximations" in the design and arriving at acceptable compromises. Accuracy of design should not be sacrificed for simplicity of the computations but they are both desirable. Thus a successful designer can be considered as the designer who makes valid approximations in design computations without loss in accuracy.



# CONCEPTS IN OPTIMIZATION



As approximations are inevitable in engineering design problems, assumptions are also necessary for design solutions within the limited time given to the designer. Usually, physical systems can be modelled mathematically only after several assumptions are made. These simplify the mathematics involved in modelling the actual physical system and make possible a practical solution.

# CONCEPTS IN OPTIMIZATION

In working on engineering design, designer can not isolate the problem from the surrounding conditions. Besides the effects of the neighbor hooding machines, there are some limitations which have become quite important in recent years. We can not use as much energy as we like and the supply of fresh water and even fresh air are limited. The environmental pollution problem should be considered anywhere on the world. Thus "the Optimum Solution" and "the Optimal Solution" may not be same for most of the problems. We may develop an ideally optimum design but it is usually never possible to produce it.

# CONCEPTS IN OPTIMIZATION

When a designer mentions Optimization, he/she actually means Maximization or Minimization of a certain **CRITERION** for a **SYSTEM**.

"**System**", in general, defines the boundaries of our concern. For a mechanical system, it is either a "machine element", or a "machine" or "plant". Generally a system does not function all alone, but works always in connection with some other system.

# CONCEPTS IN OPTIMIZATION

For example, a car is driven on a road, an engine is coupled to a gear box and gears are keyed to shafts. Thus the Shaft, the Key, the Gear, the Gearbox, the Engine and the Car can be defined as Mechanical Systems, in general. (This definition of "system" is given only for our purposes and for other fields of study, the definitions are left open to discussion). Accordingly, a **SUBSYSTEM** is any element within the system. Thus a gear box is a subsystem within a car, or car is a subsystem within the traffic.

# CONCEPTS IN OPTIMIZATION

*Subsystems of the gear* are, teeth, hub, involute curvature, keyway, bead, rim, hub bead and arm. Boundaries of the system and subsystems are defined by the design engineers according to their considerations on the problem.

If a system is optimized, the subsystems may not be optimum designs, or if all subsystems are designed as optimum, the system constructed of these subsystems may not be optimum. Even the criteria of optimization may not be the same. The system may be designed for maximum economy but the machine elements may be designed for maximum strength with weight or volume limitations.

# CONCEPTS IN OPTIMIZATION



Another basic concept in optimization is the "criterion".

The **criterion**, solely determines the set of optimum design parameters. If the criterion is changed the optimum parameters will be different. It is a reference which defines the parameters as optimum or not. Every design problem has its own criteria. A design product may be desired to be attractive and cheap. Thus Appearance must be maximized and cost will be minimized. All of the optimization methods can optimize usually one of the criteria, for optimization of several criteria there are two approaches:

# CONCEPTS IN OPTIMIZATION



i) The existing criteria can be re-defined as a new equivalent criterion. "Weighing Functions" are defined for each criterion and different criteria are summed,

ii) The design criteria are usually desirable effects. When more than one design criterion exists, it is the designer's duty to determine the "most significant" one. The "most significant criterion" (**Most Desired or Undesired**) is taken as the optimization criterion.

# CONCEPTS IN OPTIMIZATION



The choice of the most significant criterion among the existing criteria is sometimes one of the most difficult steps of the optimization process. The decision is solely up to the designer and all the responsibility is taken by him. The customer is not usually qualified to make such a decision. Few of the customers make their choice on the basis of safety for household appliances. The appearance, functioning and economy usually come first. He/She assumes that It is safe without knowing it explicitly. Since human life is under consideration in such a case, the safety of the device is one of the design criteria.



# CONCEPTS IN OPTIMIZATION

Once the designer decides on the most significant criterion, the remaining criteria should not be discarded but considered as design constraints for the problem. Design can be carried to optimize the "**most significant criterion**" while remaining within the confines of these constraints (secondary design criteria). If all of the parameters of the design can not be determined by using the "most significant criterion" then second most significant criterion should be decided and the design should be re-optimized with the already determined optimum parameters.

# CONCEPTS IN OPTIMIZATION



The design criteria are dependent on the particular design problem. Thus a spring design can be carried for minimum weight and volume for an aircraft application and the very same spring design (same spring gradient and force) can be carried for minimum or maximum free length or external diameter on a ground application. The decision will be again taken by the designer.

# CONCEPTS IN OPTIMIZATION



Optimization is an appealing field for the designer and he would like to optimize everything in his project whenever he thinks over a system. The designer should not rely on his solution during the early stages of the design process and he should not attempt at all for an optimization. The first aim in any design is certainly to satisfy the main requirements.

# CONCEPTS IN OPTIMIZATION



Thus, minimizing the weight of a sewing machine which does not sew properly is not reasonable. The elements of the machine will be changed soon, thus optimization will be a useless effort. The optimization must be carried when the whole system with all of the subsystems and elements is defined clearly and strictly for proper functioning, it is the last stage of a design process before presentation and production.

# FORMULATION OF OPTIMIZATION PROBLEMS

It is possible to develop some subjective optimization methods where engineering intuition and experience is the only tool. In this section optimization problems which can be formulated analytically will be considered.

Every optimization problem has a criterion. If all the design variables were known, it would be possible to calculate the numerical value of the criterion. The design variables (parameters) are classified as input parameters, output parameters and dummy design parameters.

# FORMULATION OF OPTIMIZATION PROBLEMS

The input parameters are independent parameters of the design. Their values can be varied freely within certain limits. The output parameters are dependent on and determined by the input parameters. The dummy parameters are used in calculating output parameters in terms of input parameters. They are used during the design optimization but do not affect the results directly. All of these design parameters define a particular set;

►  $\{x_1, x_2, x_3, \dots, x_n\}$

$$F = F(x_1, x_2, x_3, \dots, x_n)$$

# FORMULATION OF OPTIMIZATION PROBLEMS

where  $x_1, x_2, \dots, x_n$  are the design parameters, including input, output and dummy parameters. Thus dimensions, material parameters, weight, natural frequency, spring gradient are design parameters.

Once the numerical value of any set  $x_1, x_2, \dots, x_n$  is known, the design is fixed and value of the design criterion is also known. The mathematical equation which expresses the design criterion in terms of design parameters is called **CRITERION FUNCTION**. Mathematically we have;

$$F = F(x_1, x_2, x_3, \dots, x_n)$$

# FORMULATION OF OPTIMIZATION PROBLEMS

where  $x_1, x_2, \dots, x_n$  are the same parameters discussed above. The equation can be in general, in any form, linear, nonlinear, transcendental, numerical or even graphical. The form actually determines the method of solution.

For a known set of design parameters, the calculated value of the criterion function  $F$ , will be different from any other set of design parameters. For example we may have  $F_1, F_2, F_3, \dots$ , as the values of the criterion function for different sets of design parameters.



# FORMULATION Of OPTIMIZATION PROBLEMS



Certainly, one of these criterion functions will be better in a predetermined way. The aim in optimization is to find such a set of design parameters which will give the best criterion function for a predetermined purpose. The procedure to determine this set is called optimization.

Decision on the feasible sets of design parameters, are based on two groups of constraints which are derived from the requirements and limitations of the design process. The **FUNCTIONAL CONSTRAINTS** are equations which relate the design parameters to each other.

# FORMULATION OF OPTIMIZATION PROBLEMS

They are usually mathematical expressions for the laws of the nature governing engineering laws, geometric relations and similar equations. Functional constraints are written in terms of the design parameters similar to the criterion function.

$$g(x_1, x_2, x_3, \dots, x_n) = 0$$

Where  $m$  is the number of functional constraints.

# FORMULATION OF OPTIMIZATION PROBLEMS

The number of independent functional constraints is limited by the number of design parameters. If they are equal, there is only one set of parameters which satisfy the functional constraints, therefore we can not have an optimization problem. For a real solution, the number of independent functional constraints must be less than number of design parameters, i.e.,  $m < n$

# FORMULATION OF OPTIMIZATION PROBLEMS



The constraints can have two types of limits. If the limiting values can be changed for the benefit of criterion function or other less significant criteria, then it is called a **LOOSE LIMIT**. If the limiting values cannot be changed under any condition then it is a **RIGID LIMIT**. Whether a limiting value is a rigid or loose limit should be determined by the designer. This decision is reached by considering the problem statement.

# FORMULATION OF OPTIMIZATION PROBLEMS

Considering three groups of relations (criterion function, functional constraints and regional constraints) an analytical optimization problem can be formulated in the following form in its most general case.

Maximize

Minimize

$$F = F(x_1, x_2, x_3, \dots, x_n)$$

# FORMULATION OF OPTIMIZATION PROBLEMS



Subjected to

$$g_1(x_1, x_2, x_3, \dots, x_n) = 0$$

$$g_2(x_1, x_2, x_3, \dots, x_n) = 0$$

.

$$g_m(x_1, x_2, x_3, \dots, x_n) = 0$$

where  $m < n$

# FORMULATION Of OPTIMIZATION PROBLEMS



$$h_1(x_1, x_2, x_3, \dots, x_n) \geq 0$$

$$h_2(x_1, x_2, x_3, \dots, x_n) \geq 0$$

.

.

$$h_p(x_1, x_2, x_3, \dots, x_n) \geq 0$$

where  $p$  has no limit. Writing the above relations in a more compact form, we obtain;

# FORMULATION OF OPTIMIZATION PROBLEMS

Max. or Min.  $F = F(x_1, x_2, x_3, \dots, x_n)$

Subjected to

$$g_i(x_1, x_2, x_3, \dots, x_n) = 0 \quad i=1, m \quad \text{and } m < n$$

$$h_j(x_1, x_2, x_3, \dots, x_n) \geq 0 \quad j=1, p \quad \text{and } p \text{ has no limit}$$



# FORMULATION OF OPTIMIZATION PROBLEMS

The most difficult part of optimization problems is to put the problem statement in the above mathematical form. The problem must be carefully analyzed and studied so as not to exclude any of the constraints. Even if one of the constraints is not considered, the optimum solution will not be a correct or more truly, a valid one. The designer must be very careful in considering all of the related constraints.

# FORMULATION OF OPTIMIZATION PROBLEMS



Although some of the constraints are given explicitly in the problem statement, some are implicit and should be discovered by the designer.. For example, strength of the machine elements is not explicitly stated in the problems, but almost every machine element must be checked for strength. Once the problem is put in the given standard form, the rest of the work is easy. One of the optimization procedures can be applied without much difficulty. Computers can also be used after the problem is properly formulated.

End of Chapter