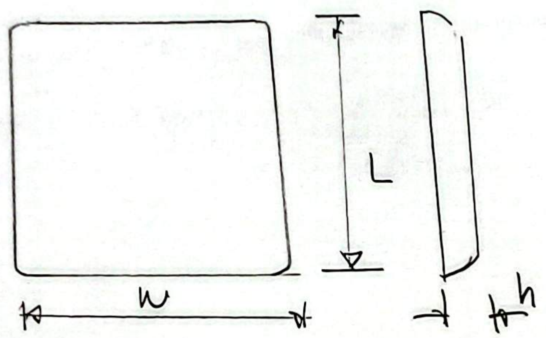
Exp. A flat rectangular tray is to be mass produced. The container must have $2000\ cm^3$ of volume. Determine the dimensions for minimum cost. Height of the tray should not exceed 2 cm and thickness of the material be at least 3 mm.



Material cost (C_m) per tray can be written as

$$C_m = V_m \gamma\, c^*_m = m^3 \left(\frac{kg}{m^3}\right)(TL/kg)$$

where V_m is the volume of the tray material, γ is specific weight, c^*_m is the unit cost of tray material (TL/kg).

Volume of the tray material (V_m) can be written approximately as;

$$V_m = wLt + 2wht + 2Lht$$

where

w = width of the tray
L = length of the tray
h = height of the tray
t = thickness of the material

$$C_m = C_m^{*} \gamma' t \,(wL + 2wh + 2Lh)$$

Volume of the tray is fixed by the problem Statement. Therefore

$$V = wLh = 2000 \text{ cm}^3$$

Also some regional constraints are introduced.

$$h \leq 2 \text{ cm}$$

$$t \geq 0.3 \text{ cm}$$

$$C_m = C_m^{*} \gamma' t \,(wL + 2wh + 2Lh)$$ is

the objective function which we seek to minimize, t, w, L, h are the parameters. The given optimization problem is written in the proper mathematical form

$$\text{Min.} \quad C_m = C_m^{*} \gamma' t \,(wL + 2wh + 2Lh)$$

$$\text{S.t.} \quad wLh - 2000 = 0$$

$$h - 2 \leq 2$$

$$t - 0.3 \geq 0$$

CamScanner ile tarandı

Exp. A cylindrical water tank is to be designed with a known and fixed volume. The weight of the empty tank should not exceed an upper limit. Formulate the problem for maximum strength.

Criterion for this problem is the maximization of strength of the tank. This is achieved either by minimizing the design strength or maximizing the factor of safety. Considering an internal pressure of p and applying simple methods of strength of materials, the criterion function can be written in either of the following forms;

$$\sigma_d = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \quad \text{or}$$

$$\sigma_d = \sqrt{\left(\frac{Pd}{2t}\right)^2 + \frac{3p^2d}{2t} + 2p^2}$$

Where σ_d is the design strength. The factor of safety n can be defined as

$$\sigma_d = \frac{\sigma_y}{n} \quad \text{or} \quad n = \frac{\sigma_y}{\sigma_d}$$

where σ_y is the yield strength of the tank's material.

So, Criterion function may be written as

Minimize $$\sigma_d = \sqrt{\left(\frac{pd}{2t}\right)^2 + \frac{3p^2d}{2t} + 2p^2}$$

or

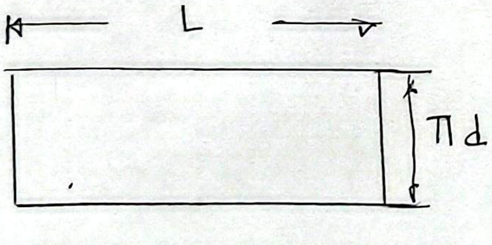
Maximize $$n = \sigma_y \left[\left(\frac{pd}{2t}\right)^2 + \frac{3p^2d}{2t} + 2p^2\right]^{-1/2}$$

Where d is the tank diameter, t is the thickness, σ_y is the yield point of the tank material. The problem has an explicit functional constraint on volume. writing in proper form;

$$V_o - \frac{\pi}{4} d^2 h = 0$$

Where V_o is known numerically.
The explicit regional constraint is on the weight.

$$\pi \gamma t d \left(\frac{d}{2} + h\right) - W_u < 0$$



$$W = (\pi d)(h)(t)\gamma + 2\left\{\frac{\pi}{4} d^2 t \gamma\right\}$$

$$W = \pi \gamma t d h + \frac{\pi}{2} d^2 t \gamma$$

Where γ is the weight density and W_u is the upper limit on the weight. Three more regional constraints can be written for this problem.

$$t > 0 \quad , \quad d > 0 \quad , \quad h > 0$$

The last inequalities should be written for all of the geometrical parameters and for non-negative physical quantities, these are called non-negativity conditions (excludes zero).

Thus, the above example can be formulated in its final form;

$$\text{Max.} \quad n = \sigma_y \left[\left(\frac{pd}{2t}\right)^2 + \frac{3p^2d}{2t} + 2p^2 \right]^{-1/2}$$

$$\text{s.t} \quad V - \frac{\pi}{4} d^2 h = 0$$

$$\pi \gamma t d \left(\frac{d}{2} + h\right) - W_u < 0$$

$$d > 0, \quad t > 0, \quad h > 0$$

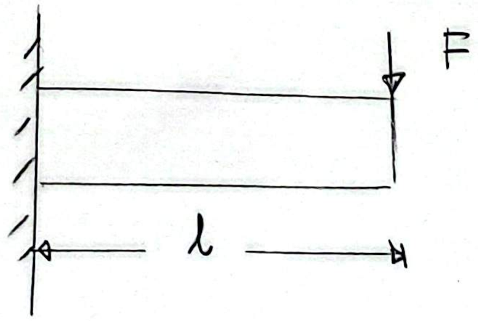
Design parameters for this problem $\sigma_y, \gamma, d, h, t$. Thus we have

$n = 5$

$m = 1$

$p = 4$

Exp. In a certain machine design problem, it is decided to use a cantilever beam as an element for energy storage. The beam cross-section can be rectangular ($b \times h$) or circular (ϕd). If it is possible, it is desired to have width b, 5 times the height h, if rectangular cross-section is selected. If circular cross-section is preferred, the diameter d should be grater than 5 mm and less than 200 mm due to space restrictions. Total weight of the beam is also restricted to 800 N, which is the max. allowable load that can be carried by the base where the beam is fixed. Also the deflection due to the weight should be less than 17.5 mm and the total deflection must be less than 100 mm. Formulate the problem for maximum energy storage capacity.



The criterion function of the problem is the mathematical expression for the stored potential energy. It has two contributions:

i) Concentrated force "F" applied at the free end of the beam. Stored energy for this load is given as

$$(PE)_F = F \cdot \Delta_F$$

where Δ_F is the deflection at the point of application due to F.

$$\Delta_F = \frac{FL^3}{3EI}$$

$$(PE)_F = \frac{F^2L^3}{3EI}$$

where L is the length of the beam, E is modulus of elasticity and I is the moment of inertia.

ii) weight of the beam. Energy equation for this case,

$$(PE)_w = \int_0^L \Delta_w \cdot w \cdot dx$$

where w is the weight per unit length, x is the coordinate axis along the beam and Δ_w is the deflection due to w.

$$(PE)_w = \int_0^L \frac{w}{24EI}(x^4 - 4L^4 + 3L)\, w\, dx$$

After integration

$$(PE)_w = \frac{w^2 L^5}{20EI}$$

Thus, total potential energy

$$(PE)_T = \frac{1}{EI}\left(\frac{1}{3}F^2L^3 + \frac{1}{20}w^2L^5\right)$$

This is the mathematical equation which is to be maximized. The governing functional constraints and regional constraints can be written properly and the standard formulation for the optimization problem can be obtained in the following form.

$$\text{Max.} \quad (PE)_T = \frac{1}{EI}\left(\frac{1}{3}F^2L^3 + \frac{1}{20}w^2L^5\right)$$

s.t.

$$\sigma_{max} - \frac{Lh}{4I}(2F + wL) = 0$$

$$w - A.\gamma = 0$$

$$W - wL = 0$$

$$\Delta_w - \frac{wL^4}{8EI} = 0$$

$$\Delta_F - \frac{FL^3}{3EI} = 0$$

$$\Delta_T - \Delta_F - \Delta_w = 0$$

$$\sigma_{all} - \frac{\sigma_y}{n} = 0$$

Rectangular section

$$I - \frac{bh^3}{12} = 0$$

$$A - bh = 0$$

$b - 5h = 0$ (Loose limit)

Circular Section

$$I - \frac{\pi d^4}{64} = 0$$

$$A - \frac{\pi d^2}{4} = 0$$

$$d - 5 > 0$$

$$d - 200 < 0$$

$$\sigma_{max} - \sigma_{all} < 0$$

$$w - 80 < 0$$

$$\Delta_w - 17.5 < 0$$

$$\Delta_T - 100 < 0$$

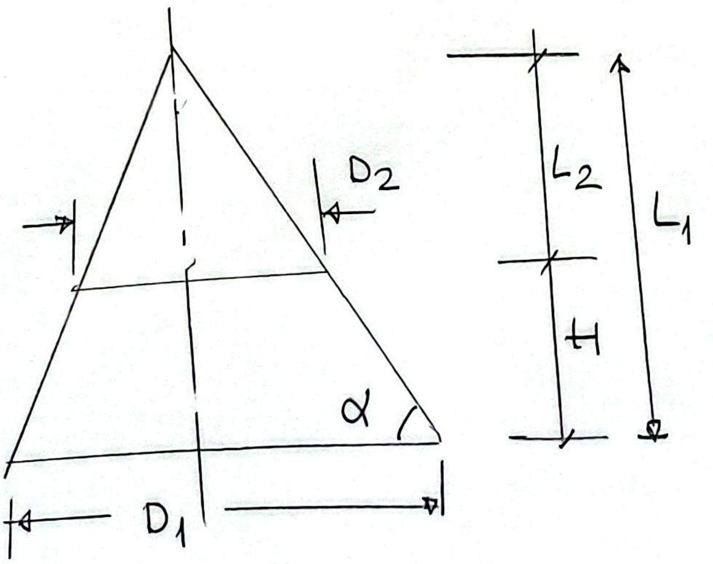
$$N - 4 > 0$$

Design parameters;

$F, W, w, L, b, h, d, A, I, \Delta_F, \Delta_w, \gamma$

σ_{all}, σ_{max}, n (That is $n=16$, $m=10$ or $m=9$, depending on rectangular or circular section and $p=5$ or 7)

Exp. It is required to design and manifacture raw material storage hoppers in truncated cone shape with 180 m^3/store capacity. The number of these stores is large so that an optimization is desired. The criterion is to minimize the ceiling area (smaller area of the truncated cone). This is because the installments will be made in a heavily snowing region, hence extra weight from the snow and ice is desired to be minimum. Due to same reasoning, the sides of the cone is required to be at greater angles than 75 degrees, or at least equal to and the weight per unit area on the floor (base) is also limited and should not exceed 5 kg/cm². The weight density of the raw material is 2 tons/m³. Assuming cost is not as important as the above stated criteria, formulate the optimization problem for determining the truncated cone dimensions.



Volume V
Unit weight γ
Floor pressure W
Ceiling area A_2

Minimize $$A_C = \frac{\pi}{4} D_2^{2}$$

Subjected to

$$\frac{\pi}{12}\left(D_1^2 L_1 - D_2^2 L_2\right) - 180 = 0$$

$$\tan\alpha - \frac{2L_1}{D_1} = 0$$

$$\tan\alpha - \frac{2L_2}{D_2} = 0$$

$$H - L_1 + L_2 = 0$$

$$W - \frac{4V\gamma}{\pi D_1^2} = 0$$

$$\alpha - 75 \geq 0$$

$$\alpha - 90 < 0$$

$$W - 5 \geq 0$$